Soluție

1.a) $B^t = A^t + A = B$,

b) Din ipoteză rezultă $A = \begin{pmatrix} 1 & x & y \\ -x & 1 & z \\ -y & -z & 1 \end{pmatrix}$, $x, y, z \in \mathbb{R}$, deci $\det(A) = 1 + x^2 + y^2 + z^2 \geq 1$.

c) Dacă $x + y = 0 \Rightarrow y = -x \Rightarrow B = x\left(A - A^t\right) \Rightarrow \det B = 0$.

2.a) $x\left(x^2 + 1\right) = 0 \Rightarrow x_1 = 0, x_2 = i, x_3 = -i$.

b) $i(p+2) + p + q - 2 = 0, p, q \in \mathbb{R} \Rightarrow p = -2, q = 4$.

$S_n = x_1^n + x_2^n + x_3^n, n \in \mathbb{N}. S_n = -pS_{n-2} - qS_{n-3}, \forall n \geq 3.$

c) $S_0 = 3, S_1 = 0, S_2 = -2p, S_3 = -3q, S_4 = 2p^2, S_5 = 5pq.$

$S_6 = -2p^3 + 3q^2, S_7 = -7p^2q.$